Inference of Schrödinger's Equation from Classical Wave Equation (1) Single Charge in Zero Potential Field

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We show that the classical wave equation for a single oscillatory charge together with the electromagnetic waves generated by it representing as a whole a de Broglie particle which may be in general traveling in a potential field V, embraces a separable component equation describing the particle dynamics. The latter is equivalent to Schrödinger's wave equation. In this first paper of a series we infer Schrödinger's wave equation for such a charge and wave system with $V = 0$.
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I Introduction

Schrödinger's wave equation [1] has proven to govern the motions of objects, in terms of observables and states in the coordinate space, in geometries comparable with their de Broglie wavelengths, and in the non-relativistic form in question here. In momentum space it has as its counterpart the Heseinberg-Born matrix formalism [2–5] and in relativistic regime the Dirac equation [6]. However, up to the present (i) the interpretation of E. Schrödinger's wave function or more generally of the quantum mechanics [7–9], and accordingly (ii) the derivation of the Schrödinger's equation have remained two major agonies of quantum mechanics in terms of transparency, unity with classical mechanics, and a workable scheme. Question (i) has summoned physicists under ardently discording schools, including among the prominent L. de Broglie [10] and E. Schrödinger's [1] phenomenological wave processes, allied by the EPR objective reality [11], W. Heisenberg [12] and M. Bohr's [13] non-deterministic hypothesis, D. Bohm's hidden variable theory [14], stochastic dynamics representation developed by a number of authors since the 50s [15,16] (see further review in [17]), and recently N. D. Mermin's objective probability [18]. Also as a textbook pedagogical illustration (e.g. [19,20]), a point-like particle has been considered as a traveling wave packet formed by a bunch of waves with slightly differing frequencies.

A derivation of Schrödinger equation, logically, is a formal mathematical implementation of the physical process on the basis of a given interpretation, if the latter does permit one. Schrödinger developed his wave equation upon his vision of the analogy between the quantum-mechanical waves and the classical mechanical waves [1] though in a phenomenological fashion, much inspired by L. de Broglie's hypothetical phase wave [10,20].

Any further effort to advance beyond Schrödinger ought in the authors' opinion *to conjoin in a physically transparent manner the observables/states of a quantum particle with the mechanical process of certain substantial entities correlated directly with the particle motion* (call this our *criterion*). As based on observations on diffraction or refraction of electrons [21,22], atoms [23] and other matter particles (see e.g. survey in [20]) the *process* of the entities associated with a particle (or particles) ought in turn to meet the requirements of (a) being extensive in space at any point in time, (b) being periodically propagated in space, and (c) having a definite wavelength Λ_d, and frequency Ω_d fulfilling de Broglie relations.

The stochastic electrodynamics approach essentially captures the random-motion nature of a particle when in a many-particle assembly, or more intrinsically as one may presume, the zero-point fluctuation of such a particle, which in principle complies to the requirements (a)–(c). Within this framework there have been several attempts to derive Schrödinger's equation [24–26,17], despite the restrictions as indicated by T. Boyer [17]. The underlying idea is that in macroscopic terms the motion of a Brownian particle is described by the diffusion equation which is formally identifiable with the Schrödinger's wave equation. Over the past three decades this framework has also instrumented fruitful explorations of a class of nonlinear Schrödinger equations [27–29] and the generalization of these and the subsequent investigations by H.-D. Doebner, G. Goldin and collaborators [30–34].

On the other hand, in the light of a scheme where the quantum variables and states describe directly the physical processes of the *internal components* of a single particle (the criterion), no successful effort going beyond L. de Broglie's hypothetical phase wave has been made prior to our recent work introduced fully in Ref. [35]. By the unification scheme proposed in [35] (brief outline in Ref. [36]), a simple or basic material particle (like an electron, proton, positron, muon, and so forth), briefly, is formed by an oscillatory elementary charge ($-e$ or $+e$) together with the electromagnetic waves generated by it in the vacuum. We showed in Ref. [35] through the solutions for the Newtonian-Maxwellian equations of motion, that such a particle has the overall observational properties including charge, mass, spin, de Broglie wave properties, the obedience of de Broglie relations and of energy-mass equivalence relation, and all of the relativistic qualities conforming to observations. Here the *criterion* and the requirements (a)–(c) are met.

In the present series of papers we derive in a self-contained fashion the Schrödinger wave equation for an aforesaid particle starting from the classical wave mechanics description of it. In a separate publicized short paper we show that between such particles in a dielectric medium [35] (2004c), there presents a universal gravity. Readers having not been exposed to the other justified aspects in [35] are hoped to also find the paper of value in two respects: (1) The oscillatory charge and the electromagnetic waves predicted by Maxwellian electrodynamics are together a simple system involving but a transparent and self-contained physics; and (2) the system can therefore find use, letting be in the restricted contexts of properties as justified in this paper, in quantum-mechanical studies such as nano science and quantum information among others.

In this paper we discuss the case of having a zero applied potential, $V = 0$, to focus on aspects other than those under the influence of V. The transformation of V, acting directly on the particle's charge, to acting on the particle's wave is non-trivial. To not complicate the discussion we also assume the particle, when in motion of velocity v, is within a non relativistic or classic-velocity regime, expressed formally by $(v/c)^2 \to 0$; a generalization of the derivation to the relativistic or unclassic-velocity form is in principle straightforward.

II Classical wave equation for particle's total wave

We consider a single oscillatory charge q firstly localized at X in a one-dimensional box along X-axis, in a zero potential field $V = 0$. According to Maxwellian electromagnetism, owing to its oscillation assumed transverse to the X-axis the charge generates electromagnetic waves in both $+X$- and $-X$-directions. The wave process may be described as conventionally by the electric field displacement $\mathbf{E}$ or magnetic field $\mathbf{B}$, or by a counterpart of $\mathbf{E}$ or $\mathbf{B}$ representing the corresponding mechanical scheme; the latter is not of direct concern of this paper so is not to be introduced.

With either of the representations in mind, we may rather generally describe the periodic process with a dimensionless wave displacement $\varphi^j_{\scriptscriptstyle K}(X,T)$ with $j = \dagger, \ddagger$ indicating propagations in $+X$-, $-X$-directions. The two functions, describing each an electromagnetic wave propagated along a one-dimensional path in the uniform vacuum, are as based on optical experimental physics plane waves:

$$\varphi^j_{\scriptscriptstyle K}(X,T) = C_{\scriptscriptstyle K} e^{\pm i(KX \mp \mathit{\Omega} T + \alpha^j)} \tag{1}$$

with $C_{\scriptscriptstyle K}$ being the displacement amplitude; $\varphi^j_{\scriptscriptstyle K}$ is propagated in the vacuum with the speed of light, c. Here K and $\mathit{\Omega}(= 2\pi\mathcal{V}) = cK$ are the wavevector and angular frequency of the wave, or prefixed with a "normal mode" in considering the correspondence of the waves to normal mode waves in the solution for a mechanical wave (see also below). Either in making analogy to an ordinary mechanical wave, or directly in terms of $\mathbf{E}$ or $\mathbf{B}$ (see e.g. [37]), the corresponding wave displacement as of (1) is solutions to the classical wave equation

$$c^2 \frac{d^2 \varphi^j_{\scriptscriptstyle K}}{dX^2} = \frac{d^2 \varphi^j_{\scriptscriptstyle K}}{dT^2} \tag{2}$$

where $j = \dagger, \ddagger$. Alternatively, (2) can also be set up based on the argument that, as being in direct proportion to the electromagnetic energy flux transmitted with it in a non-absorbing vacuum, $\varphi^j_{\scriptscriptstyle K}(X,T)$ must therefore satisfy the time dependent Laplacian equation characterizing the conservation of energy, which is (2). In an elaborate treatment where a corresponding mechanical scheme is incorporated to the wave motion, (2) can be directly set up based on Newton's laws of motion, and (1) is given as a solution [35].

In line with the particle formation scheme of Ref. [35,36] outlined earlier, we shall refer to the oscillatory electric charge, of a charge q (which may be called a vaculeon in the case of an elementary charge) and oscillatory mechanical energy E, as a whole as a particle. The oscillation represents then an internal process of the particle. When regarding the particle's external process only, e.g. its translational motion at a velocity v, we need then only to represent this external process without regarding the internal ones. The two opposite traveling waves $\varphi^{\dagger}_{K}(X,T)$ and $\varphi^{\ddagger}_{K}(X,T)$ of (1) are alternative representations of the particle's internal charge oscillation, and represent therefore the component (normal mode) waves of the particle. The electromagnetic waves as described by (1) have according to Planck a mechanical energy

$$\mathsf{E} = \hbar \Omega \tag{3}$$

For the waves being here its component waves, E of (3) amounts thereby in turn to (here all of) the internal energy of the particle. Then, from Einstein's mass energy equivalence postulate, E is related to the particle's rest mass M by $\mathsf{E} = Mc^2$; this and (3) give

$$\hbar \Omega = Mc^2 \tag{4}$$

In the systematic framework of [35], (3) and (4) are directly given as a Newtonian-Maxwellian solutions for particle formation.

Consider next our particle as defined above is set in motion at velocity v firstly in $+X$-direction; $(v/c)^2 \to 0$ as specified earlier. Owing to its charge (source) motion the two monochromatic component waves of the single frequency Ω must now be subject to a Doppler effect, and be differentiated into an approach- and a recede- Doppler wave, $\varphi^{\dagger}_{K^{\dagger}}$ and $\varphi^{\ddagger}_{K^{\ddagger}}$. Their wave equations corresponding to (2) accordingly become:

$$\begin{aligned} \frac{\partial^2 \varphi^{\dagger}_{K^{\dagger}}(X,T)}{\partial X^2} &= \frac{1}{c^2}\frac{\partial^2 \varphi^{\dagger}_{K^{\dagger}}(X,T)}{\partial T^2} \\ \frac{\partial^2 \varphi^{\ddagger}_{K^{\ddagger}}(X,T)}{\partial X^2} &= \frac{1}{c^2}\frac{\partial^2 \varphi^{\ddagger}_{K^{\ddagger}}(X,T)}{\partial T^2} \end{aligned} \tag{5}$$

Being bound within walls assumed here non-absorbing, the particle traveling to the right will upon arriving at the right wall be reflected from the wall, and travel to the left during which it generates waves denoted by $\varphi^{\dagger *}_{K^{\dagger}}(X,T)$ and $\varphi^{\ddagger *}_{K^{\ddagger}}(X,T)$. The wave equations for the latter write similarly as (5). The two sets of component waves, $\varphi^{\dagger}_{K^{\dagger}}$,$\varphi^{\ddagger}_{K^{\ddagger}}$ and $\varphi^{\dagger *}_{K^{\dagger}}$,$\varphi^{\ddagger *}_{K^{\ddagger}}$, produced by the forwarding and reflected charges, will be each repeatedly reflected between the boundaries, prevailing across the entire one-dimensional box simultaneously at any moment in time. These thereby superimpose onto each other and the sum of the respective functions, supposing with the respective relative phases properly contained, giving a total wave displacement:

$$\psi(X,T) = \widetilde{\psi} + \widetilde{\psi}^* = C\{[\varphi^{\dagger}_{K^{\dagger}}(X,T) + \varphi^{\ddagger}_{K^{\ddagger}}(X,T)] + [\varphi^{\dagger *}_{K^{\dagger}}(X,T) + \varphi^{\ddagger *}_{K^{\ddagger}}(X,T)]\} \tag{6}$$

normalized here with the normalization factor C. Adding in turn together the two equations of (5), and the corresponding two equations for the reflected charge, we get

$$\frac{\partial^2 C[(\varphi^{\dagger}_{K^\dagger}+\varphi^{\ddagger}_{K^\ddagger})+(\varphi^{\dagger *}_{K^\dagger}+\varphi^{\ddagger *}_{K^\ddagger})]}{\partial X^2}=\frac{1}{c^2}\frac{\partial^2 C[(\varphi^{\dagger}_{K^\dagger}+\varphi^{\ddagger}_{K^\ddagger})+(\varphi^{\dagger *}_{K^\dagger}+\varphi^{\ddagger *}_{K^\ddagger})]}{\partial T^2}. \tag{a}$$

With (6) in (a), we have that the classical wave equation for the particle's total wave displacement is

$$c^2\frac{\partial^2\psi(X,T)}{\partial X^2}=\frac{\partial^2\psi(X,T)}{\partial T^2} \tag{7}$$

For the component waves of (6) as given by (1) and differing from one another only in their relative phases, as may be readily completed by the reader (or see [35]) ψ can be explicitly obtained from a direct superposition, firstly within each set, to yield two opposite-traveling beat waves $\widetilde{\psi}$ and $\widetilde{\psi}^*$ of frequency $\Omega+\Omega_d\approx\Omega$ assuming $\Omega_d<<\Omega$, and phase velocity $d\Omega/dK_d=c^2/v$. The two beat waves superpose in turn into a standing beat wave described by:

$$\psi(X,T)=4CC_{K}\cos(KX)\sin(K_dX)e^{i(\Omega+\Omega_d)T}, \tag{8}$$

Where K and Ω are the undisplaced (i.e. not being Doppler-displaced) wavevector and angular frequency of the particle's component waves as before; and

$$K_d=\left(\frac{v}{c}\right)K \tag{9}$$

and

$$\Omega_d=K_dv=\left(\frac{v}{c}\right)^2\Omega \tag{10}$$

are the mean Doppler displacements of wavevectors and angular frequencies, respectively, as can be straightforwardly obtained based on the Doppler equations for wavelengths $\Lambda^\dagger=\Lambda(1-v/c)$ and $\Lambda^\ddagger=\Lambda(1+v/c)$. We also know from L. de Broglie that the de Broglie variables of a particle are purely as a result of the particle motion, here the translation of velocity v. From the above connection, one may plausibly identify K_d as the particle's de Broglie wavevector and Ω_d the de Broglie angular frequency. The corresponding de Broglie wavelength and period are $\Lambda_d=2\pi/K_d=\left(\frac{c}{v}\right)\Lambda$ and $\Gamma_d=2\pi/\Omega_d=\left(\frac{v}{c}\right)^2\Gamma$, where $\Lambda=2\pi/K$ and $\Gamma=2\pi/\Omega$. A profound justification of the above assignment is given in [35].

Actually, based on his famous hypothetical phase wave of frequency $\Omega/2\pi$ and phase velocity $\mathsf{w}=c^2/v$ he assumed to represent the internal process of a "particle", L. de Broglie [10] obtained his de Broglie wavelength to be $\Lambda_d=\left(\frac{c}{v}\right)\frac{\mathsf{w}}{\Omega/2\pi}$, or wavevector $K_d=2\pi/\Lambda_d=\left(\frac{v}{c}\right)\frac{\Omega/2\pi}{\mathsf{w}}=\left(\frac{v}{c}\right)K$ which is identical to (9). This hypothetical phase wave is seen to correspond precisely to our traveling beat waves $\widetilde{\psi}$ and $\widetilde{\psi}^*$.

III Dimension transformation

From inspecting (8) we see that the total particle dynamics involves two distinct wave processes—an undisplaced (normal mode) wave motion assocated with φ and a de Broglie particle wave motion associated with K_d, Ω_d. The two processes as given in (7) are apparently not mutually separated. However, (7) has an intuitively, and also physically arguably undesirable dimension of "force per unit mass of the elastic medium propagating the wave per displacement amplitude". As an attempt to decouple the two wave processes, we below transform equation (7) to having a dimension of "energy". We start by utilizing a first specific feature of the function of (8), namely that ψ may be written as a product of two functions:

$$\psi(X,T) = C\psi_{K,K_d}(X)e^{\imath(\Omega+\Omega_d)T} \tag{11}$$

Where

$$\psi_{K,K_d}(X) = 4C_K \cos(KX)\sin(K_d X) \tag{12}$$

Furthermore, $\psi(X,T)$ and its first and second time-derivatives have the following recursive relations:

$$\frac{\partial\psi(X,T)}{dT} = i(\Omega+\Omega_d)\psi_{K,K_d}(X)e^{i(\Omega+\Omega_d)T} = i(\Omega+\Omega_d)\psi(X,T) \tag{13}$$

$$\frac{\partial^2\psi(X,T)}{\partial T^2} = -(\Omega+\Omega_d)^2\psi_{K,K_d}(X)e^{i(\Omega+\Omega_d)T} = i(\Omega+\Omega_d)\frac{\partial\psi(X,T)}{\partial T} \tag{14}$$

Substituting (14) into the right-hand side of (7), rearranging, and multiplying both sides with the Dirac constant $\hbar = h/2\pi$, h being Planck constant, we have

$$\hbar c^2\frac{\partial^2\psi(X,T)}{\partial X^2} = i\hbar(\Omega+\Omega_d)\frac{\partial\psi(X,T)}{\partial T} \tag{7a}$$

(7a) has the desired energy dimension in differential forms: On the right-hand side, $\hbar(\Omega+\Omega_d)$ has the dimension of energy, and $\frac{\partial\psi}{\partial T}$ time rate; the term $\hbar(\Omega+\Omega_d)\frac{\partial\psi}{\partial T}$ as a whole has a dimension of the time rate of energy, hence power. The left-hand side has the dimension of second spatial variation of energy.

IV Decoupling of two wave processes A. Wave equation

The dynamics of a single free particle is in classical terms represented by the particle's translational kinetic energy $\mathsf{E}_v = \frac{1}{2}Mv^2$ and linear momentum Mv. The very dynamics is in quantum physics characterized by the de Broglie variables, which is on the right-hand side of (7a) apparently represented by the second term containing Ω_d. The first term describes accordingly the undisplaced normal mode wave dynamics, with Ω being a variable for the classic-velocity regime here.

The left-hand side of (7a) must accordingly also contain a component of the particle dynamics and this ought to be contained in c^2 which we show below to be indeed the case. As based on Einstein's phenomenological postulate, or as is given directly from Newtonian-Maxwellian solution for particle formation [35], c^2 is related to the particle's unclassic-velocity mass indicated by the corresponding lowercase-letter m, as:

$$c^2 = \frac{\hbar\omega}{m} = \frac{\hbar\gamma\Omega}{m} \tag{16}$$

Where

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}} \tag{17}$$

For $(v/c)^2 \to 0$:

$$c^2 \approx \frac{\hbar\Omega}{M}, \tag*{(16)$'$}$$

with M being the particle's rest mass as before. The approximation (16)$'$ is factually to a high degree of accuracy when expressing the particle's total rest energy [35]. But this approximation is inevitably a poor one for the particle (center-of-mass) dynamics in question here, since the kinetic energy (being a classic-velocity quantity) of the latter is itself of order v^2, and is always of first-order significance no matter how small $(v/c)^2$ is. Hence, to ensure an exact representation of the particle dynamics, we reset all variables of equation (7a) to their unclassic-velocity forms expressed in the corresponding lowercase letters, separate out the particle center-of mass term on the left-hand side, and then take the classic-velocity limit. In doing so, (7a) firstly writes as:

$$\frac{\hbar^2\omega}{m}\frac{\partial^2\psi(X,T)}{\partial X^2} = i\hbar(\omega+\omega_d)\frac{\partial\psi(X,T)}{\partial T} \tag{7b}$$

Here, as with the frequency of (16), as according to Einstein's phenomenological equation, and as is in [35] directly obtained from Newtonian-Maxwellian solution for particle formation, m and M are related by:

$$m = \gamma M \tag{18}$$

Combining (18) with (4) and in turn with (10) further gives

$$\omega = \gamma\Omega \quad (19) \qquad\qquad \omega_d = \gamma\Omega_d \quad (20)$$

With (18) and (17), $1/m$ may be expanded to:

$$\frac{1}{m} = \frac{1}{\gamma M} = \frac{1}{M}\sqrt{1-\left(\frac{v}{c}\right)^2} = \frac{1}{M}\left[1-\frac{1}{2}\left(\frac{v}{c}\right)^2 - \frac{1\cdot 3}{2\cdot 4}\left(-\left(\frac{v}{c}\right)^2\right)^2 + \ldots\right]$$

Sorting gives

$$\frac{1}{m} = \frac{1}{M} - \frac{\mathsf{E}_v}{M^2c^2}\left[-\frac{3}{4}\left(\frac{v}{c}\right)^2 + \ldots\right] \tag{21}$$

Where $\mathsf{E}_v = \frac{1}{2}Mv^2$ is the particle's translational kinetic energy in the classic-velocity limit as before. Hence the proper classic-velocity limit of (21) in respect to the particle's total dynamics is

$$\lim_{(v/c)^2\to 0} \frac{1}{m} = \frac{1}{M} - \frac{\mathsf{E}_v}{M^2c^2} \lim_{(v/c)^2\to 0} \frac{1}{m}\left[-\frac{3}{4}\left(\frac{v}{c}\right)^2 + \ldots\right] = \frac{1}{M} - \frac{\mathsf{E}_v}{M^2c^2}$$

That is, to a good approximation: $\frac{1}{m} = \frac{1}{M}\left[1 - \frac{1}{2}\left(\frac{v}{c}\right)^2\right]$. Multiplying its both sides with $\hbar\omega$ we have

$$\frac{\hbar\omega}{m} = \frac{\hbar\omega}{M}\left[1 - \frac{1}{2}\left(\frac{v}{c}\right)^2\right] \tag{22}$$

Substituting (22) into the left-hand side of (7b) we have

$$\frac{\hbar^2\omega}{M}\left[1 - \frac{1}{2}\left(\frac{v}{c}\right)^2\right]\frac{\partial^2\psi}{\partial X^2} = i\hbar(\omega + \omega_d)\frac{\partial\psi}{\partial T} \tag{7c}$$

We now take the classic-velocity limit of (7c); this for the individual variables amounts to taking the limits: $\lim_{(v/c)^2\to 0}\omega = \lim_{(v/c)^2\to 0}(\gamma\varOmega) = \varOmega$ and $\lim_{(v/c)^2\to 0}\omega_d = \lim_{(v/c)^2\to 0}(\gamma\varOmega_d) = \varOmega_d$, where ω and ω_d being given by (19) and (20). With the two last results, we have the classic-velocioty limit of (7c)

$$\frac{\hbar^2\varOmega}{M}\left[1 - \frac{1}{2}\left(\frac{v}{c}\right)^2\right]\frac{\partial^2\psi}{\partial X^2} = i\hbar(\varOmega + \varOmega_d)\frac{\partial\psi}{\partial T} \tag{7d}$$

Substituting into (7d) with (10) for $(v/c)^2$, we have

$$\frac{\hbar^2}{M}\left[\varOmega - \frac{1}{2}\varOmega_d\right]\frac{\partial^2\psi}{\partial X^2} = i\hbar(\varOmega + \varOmega_d)\frac{\partial\psi}{\partial T} \tag{7e}$$

In (7e), we see that the first term on either side is an energy related to the normal mode wave process, whereas the second term is one related to the de Broglie wave process.

V Decoupling of two wave processes B. Wave function

We now exploit a second feature in $\psi(X,T)$ of (11), namely that it can write as a product of two mutually orthogonal functions

$$\varPhi(X,T) = 4C_K\cos(KX)e^{\imath\varOmega T+\alpha_1} \tag{23}$$

and

$$\varPsi(X,T) = C\sin(K_dX)e^{\imath\varOmega_d T+\alpha_2}. \tag{24}$$

That is

$$\psi(X,T) = \varPhi(X,T)\varPsi(X,T). \tag{11$'$}$$

Where α_1 and α_2 are the initial phases of the two waves and are uncorrelated. For use below we further write down the time-derivatives of (23) and (24) to be:

$$\frac{1}{i}\frac{\partial \Phi}{\partial T} = \Omega\Phi \qquad (25) \qquad\qquad \frac{1}{i}\frac{\partial \Psi}{\partial T} = \Omega_d \Psi \qquad (26)$$

It is easy to identify that $\Phi(X,T)$ describes the undedisplaced total normal mode wave displacement and $\Psi(X,T)$ the de Broglie wave displacement of the particle, each being dimensionless here. Substituting $(11)'$ into (7e), expanding the derivatives, and multiplying in all the terms, we get

$$\begin{aligned} &\frac{\hbar^2 \Omega\Phi}{M}\frac{\partial^2 \Psi}{\partial X^2} + \frac{\hbar^2 \Omega\Psi}{M}\frac{\partial^2 \Phi}{\partial X^2} - \frac{\hbar^2 \Omega_d\Phi}{2M}\frac{\partial^2 \Psi}{\partial X^2} - \frac{\hbar^2 \Omega_d\Psi}{2M}\frac{\partial^2 \Phi}{\partial X^2} \\ &= i\hbar\Omega\Phi\frac{\partial \Psi}{\partial T} + i\hbar\Omega\Psi\frac{\partial \Phi}{\partial T} + i\hbar\Omega_d\Phi\frac{\partial \Psi}{\partial T} + i\hbar\Omega_d\Psi\frac{\partial \Phi}{\partial T} \end{aligned} \qquad (7f)$$

Substituting with (25) and (26), the first and the fourth term on the left and right side of (7f) become respectively $\frac{\hbar^2 \Omega\Phi}{M}\frac{\partial^2 \Psi}{\partial X^2} = \frac{\hbar^2}{M}\frac{\partial \Phi}{i\partial T}\frac{\partial^2 \Psi}{\partial X^2}$, $-\frac{\hbar^2 \Omega_d\Psi}{2M}\frac{\partial^2 \Phi}{\partial X^2} = -\frac{\hbar^2}{2M}\frac{\partial \Psi}{i\partial T}\frac{\partial^2 \Phi}{\partial X^2}$, $i\hbar\Omega\Phi\frac{\partial \Psi}{\partial T} = \hbar\frac{\partial \Phi}{\partial T}\frac{\partial \Psi}{\partial T}$, and $i\hbar\Omega_d\Psi\frac{\partial \Phi}{\partial T} = \hbar\frac{\partial \Psi}{\partial T}\frac{\partial \Phi}{\partial T}$. Each of these involves a cross-term product of the two wave processes, in terms of the derivative of Φ and that of Ψ with respect to either T or (twice) X; the two derivatives are as seen from their expressions above also orthogonal to one another, as with their original functions. Two mathematically orthogonal functions correspond in physical terms to two mutually uncorrelated processes, here the derivatives of the normal mode wave and the particle wave, which being by virtue of their wave nature (each with a random phase) statistical processes. It thus follows that these terms, i.e. the first and fourth terms on each side of (7f), must on average make each a zero contribution to the stationary-state energy equation and therefore should drop out. Multiplying each of the remaining terms with $1/\Psi\Phi$, (7f) then becomes

$$\frac{\hbar^2 \Omega}{M}\frac{1}{\Phi}\frac{\partial^2 \Phi}{\partial X^2} - \frac{\hbar^2 \Omega_d}{2M}\frac{1}{\Psi}\frac{\partial^2 \Psi}{\partial X^2} = i\hbar\Omega\frac{1}{\Phi}\frac{\partial \Phi}{\partial T} + i\hbar\Omega_d\frac{1}{\Psi}\frac{\partial \Psi}{\partial T} \qquad (7g)$$

VI Schrödinger equation with zero potential

A single particle in a one-dimensional box (of non-absorbing walls) with a zero potential field and having no contact with external environment describes an isolated system; and its total dynamics is hence an adiabatic process. That the particle's two constituent wave processes are uncorrelated then implies in energy terms that, any local and instantaneous variations (oscillations) in energy which being associated with wave displacements, must occur within the respective wave processes. By this virtue, the energy equation

(7g) for the total wave at any chosen location and time, must therefore be necessarily guaranteed by the energy equations for the respective individual waves:

$$\frac{\hbar^2 \Omega}{M} \frac{1}{\Phi} \frac{\partial^2 \Phi}{\partial X^2} = i\hbar\Omega \frac{1}{\Phi} \frac{\partial \Phi}{\partial T} \tag{b}$$

$$-\frac{\hbar^2 \Omega_d}{2M} \frac{1}{\Psi} \frac{\partial^2 \Psi}{\partial X^2} = i\hbar\Omega_d \frac{1}{\Psi} \frac{\partial \Psi}{\partial T} \tag{c}$$

Eliminating the respective common factors on both sides, Ω/Φ in (b) and Ω_d/Ψ in (c), sorting, we thus finally obtain (b)–(c) with the more desirable appearance:

$$\frac{\hbar^2}{M} \frac{\partial^2 \Phi(X,T)}{\partial X^2} = i\hbar \frac{\partial \Phi(X,T)}{\partial T} \tag{27}$$

(SWE-1) :

$$-\frac{\hbar^2}{2M} \frac{\partial^2 \Psi(X,T)}{\partial X^2} = i\hbar \frac{\partial \Psi(X,T)}{\partial T} \tag{28}$$

We readily recognize that (28) is equivalent to the Schrödinger equation in a zero potential field.

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